Also solved by H. L. OLSON, ALBERT N. NAUER, JOSEPH B. REYNOLDS, C. E. FLANAGAN, G. PAASWELL, and HERBERT N. CARLETON.

2684 [March, 1918]. Proposed by B. J. BROWN, Kansas City, Missouri.

Find the locus of the center of a conic passing through four fixed points.

I. SOLUTION BY H. D. THOMPSON, Princeton University.

This is an exercise given in books on coördinate geometry.

Take the x-axis through two of the four points, and the y-axis, oblique, through the other two. Let $1/l$ and $1/l'$ be the abscissas of the two points on the x-axis and $1/m$ and $1/m'$, the ordinates of the two points on the y-axis. Then $lx + my - 1 = 0$ and $l'x + m'y - 1 = 0$ form another pair of lines, containing the four points in pairs. All conics through the four points are given by $\lambda xy + (lx + my - 1)(l'x + m'y - 1) = 0$, when λ is the parameter of the system.

The center of a representative conic is given by $2ll'x + (lm' + l'm)y - (l + l') + \lambda y = 0$ and $(lm' + l'm)x + 2mm'y - (m + m') + \lambda x = 0$. Eliminating λ, the locus of the center is $2ll'x^2 - 2mm'y^2 - (l + l')x + (m + m')y = 0$, a conic through the origin. The center of the locus is $\{1/4(1/l' + 1/l), 1/4(1/m' + 1/m)\}$. As any pair of opposite sides of the complete quadrilateral with the original four points as vertices can be taken as the axes, the locus of the centers will pass through the three points of intersection of opposite sides of the complete quadrilateral.

The locus is an hyperbola when $ll'mm'$ is positive, that is, when the original four points may be taken as the vertices of a convex polygon; and it is in this case only that the original conic may be a parabola (two).

II. SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

The equation of a conic passing through the four given points $\pm \alpha_1$, $\pm \beta_1$, $\pm \gamma_1$, trilinear coördinates being used, is of the form

$$l_1\alpha^2 + m_1\beta^2 + n_1\gamma^2 = 0 \tag{1}$$

with the condition

$$l_1\alpha_1^2 + m_1\beta_1^2 + n_1\gamma_1^2 = 0. \tag{2}$$

The coördinates of the center of (1) are given by

$$\frac{l_1\alpha}{a} = \frac{m_1\beta}{b} = \frac{n_1\gamma}{c}, \tag{3}$$

a, b, c, being the sides of the fundamental triangle.

Eliminating l_1, m_1, n_1 from (3) and (2), we have

$$\frac{a\alpha_1^2}{\alpha} + \frac{b\beta_1^2}{\beta} + \frac{c\gamma_1^2}{c} = 0, \tag{4}$$

the required locus.

This is the nine-point conic of the quadrilateral whose vertices are the four given points.

Also solved by PAUL CAPRON and ELIJAH SWIFT.

2685 [March, 1918]. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

A particle is describing an ellipse of eccentricity $\sqrt{2/3}$ as a central orbit about a focus when the attracting force suddenly becomes repulsive without changing its magnitude and the particle begins to describe an equilateral hyperbola; find where the change occurred and the angle that the major axis of the new orbit makes with that of the old orbit.

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

The focal equation of the ellipse is

$$p_1^2 = \frac{a_1^2(1 - e_1^2)r_1}{2a_1 - r_1} \tag{1}$$

and of the hyperbola,

$$p_2^2 = \frac{a_2^2(e_2^2 - 1)r_2}{2a_2 + r_2} \tag{2}$$